

Sets Functions Quiz

1. Consider the set $A = \{\{1, 2\}, 1, 2\}$.

(a) **(1 point)** How many elements does A have?

Answer: A has 3 elements.

(b) **(4 points)** The power set $\mathcal{P}(A)$ is the set of all subsets of A . What is $\mathcal{P}(A)$ in this case?

Answer: The power set

$$\mathcal{P}(A) = \{\{\{1, 2\}\}, \{1\}, \{2\}, \{\{1, 2\}, 1\}, \{\{1, 2\}, 2\}, \{\{1, 2\}, 1, 2\}, \{1, 2\}, \{\}\}$$

2. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Define a function $f : A \rightarrow B$ via $f(x) = x^2$. In other words

$$f(1) = 1, \quad f(2) = 4, \quad f(3) = 9.$$

(a) **(2 points)** Is the function injective? If no, can you suggest a modification that makes it injective?

Answer: The function is injective.

(b) **(2 points)** Is the function surjective? If no, can you suggest a modification that makes it surjective?

Answer: The function is not surjective. Consider $C = \{1, 4, 9\} \subset B$. Then $f : A \rightarrow C$ given by $f(x) = x^2$ is surjective.

(c) **(3 points)** Define a square root \mathcal{R} from B to A as follows:

$$9\mathcal{R}3, \quad 4\mathcal{R}2 \quad 1\mathcal{R}1.$$

i. What is the domain of \mathcal{R} ?

Answer: The domain of \mathcal{R} is $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

ii. What is the codomain of \mathcal{R} ?

Answer: The codomain of \mathcal{R} is $A = \{1, 2, 3\}$.

iii. Is \mathcal{R} a function? Explain why.

Answer: \mathcal{R} is not a function because not every element in the domain has an image. For example $2, 3, 5, 6, 7, 8 \in B$ but they are not mapped to any element in A .

3. Let $[n] = \{1, 2, \dots, n\}$ be the set of first n positive integers. We define a function $f : [n] \rightarrow [n]$ by

$$f(x) = n - x + 1.$$

(a) Draw the function diagram (points and arrows) for f when $n = 5$. Label the codomain and the domain. **(4 points)**

Answer:

(b) For a general n , is the function bijective? **(4 points)**

Answer: The function is bijective.

- If the answer is **Yes**, what is the formula for the inverse?

Answer: The formula for the inverse is $n - x + 1$.

- If the answer is **No**, what fails? Injectivity or Surjectivity or both?