

**Mathematical Induction Quiz**

Time 30 mins

1. For all integers  $n \geq 1$  prove by Mathematical Induction that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$$

(a) (2 points) Base case:

(b) (8 points) Inductive case:

2. Suppose the numbers  $a_n$  are defined inductively by

$$a_1 = 1, \quad a_2 = 2, \quad a_3 = 3, \quad \text{and} \quad a_n = a_{n-1} + a_{n-2} + a_{n-3} \quad \text{for } n \geq 4.$$

(a) (2.5 points) Compute the first 8 terms of the sequence, ie compute  $a_4, a_5, a_6, a_7, a_8$

i.  $a_4$

ii.  $a_5$

iii.  $a_6$

iv.  $a_7$

v.  $a_8$

(b) Prove using Mathematical Induction that  $a_n < 2^n$  for every positive integer  $n$ .

i. (2.5 points) Base case:

ii. (5 points) Inductive case: