Chess Problems

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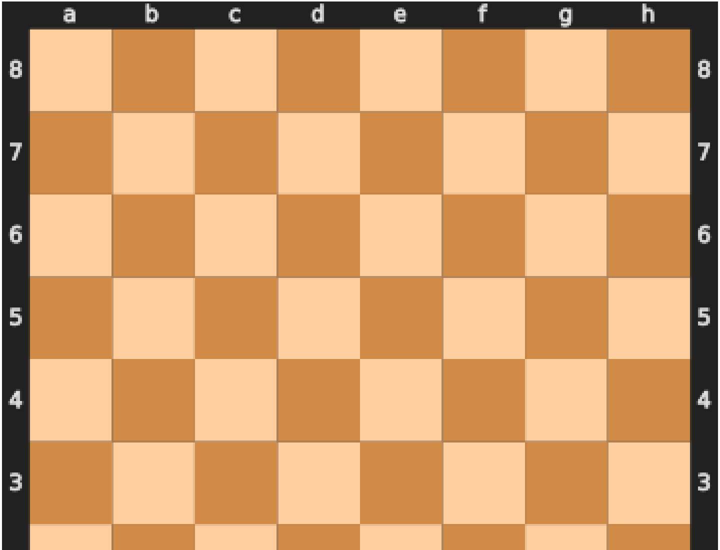
Lecture 5: Discrete Math Modelling

Chess Problems

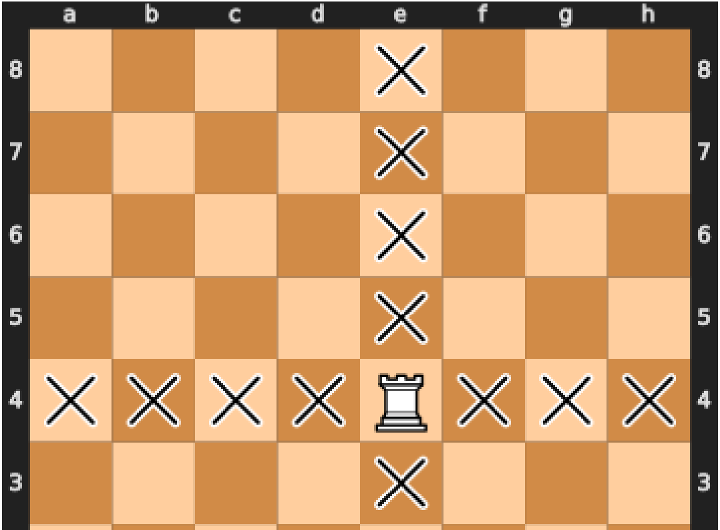
- Today we will discuss Chess Problems.
- More specifically we will discuss non-attacking problems.
- Lets begin.



Non-attacking Rooks

Place the most number of non-attacking rooks on a chess board.

- Idea: If rook is place at (i, j) then another rooks cannot be placed at the same row/column.



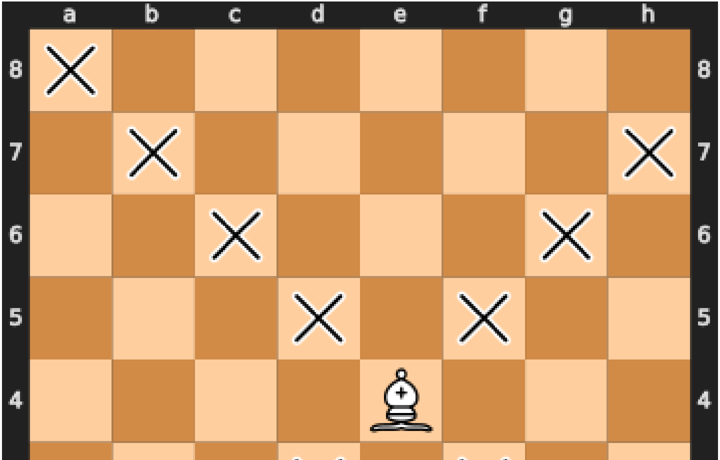
Answer:

- Answer: Let $x_{ij} = 1$ if rook is placed at row i, column j and 0 otherwise. Then we want to
- maximize $\sum_{ij} x_{ij}$ subject to
- $\sum_i x_{ij} \leq 1$ non attacking on rows.
- $\sum_j x_{ij} \leq 1$ non attacking on columns.

Non-attacking Bishops

How do we translate the same if we want non-attacking bishops instead of rooks.

- Idea: Bishops attack across diagonals $(i + j = k)$ or anti-diagonals $(i - j = k)$. Note that $2 \leq i + j \leq 16$ and $-7 \leq i - j \leq 7$.

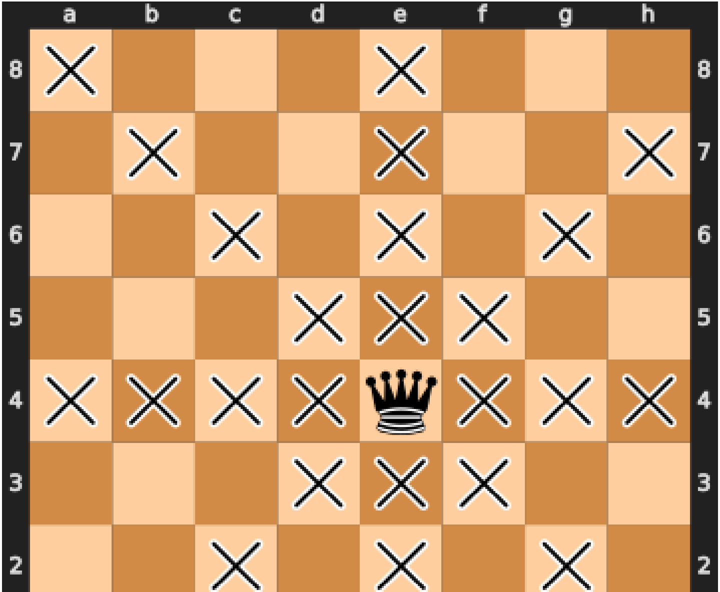


Answer:

- Answer: Let $x_{ij} = 1$ if bishop is placed at (i, j) and 0 otherwise. We want to
- maxmimize $\sum_{ij} x_{ij}$ subject to
- $\sum_{i+j=k} x_{ij} \leq 1$ for $k = 2, \ldots, 16$.
- $\sum_{i-j=k} x_{ij} \leq 1$ for $k = -7, \ldots, 7$.

Question 3: Non-attacking Queens.

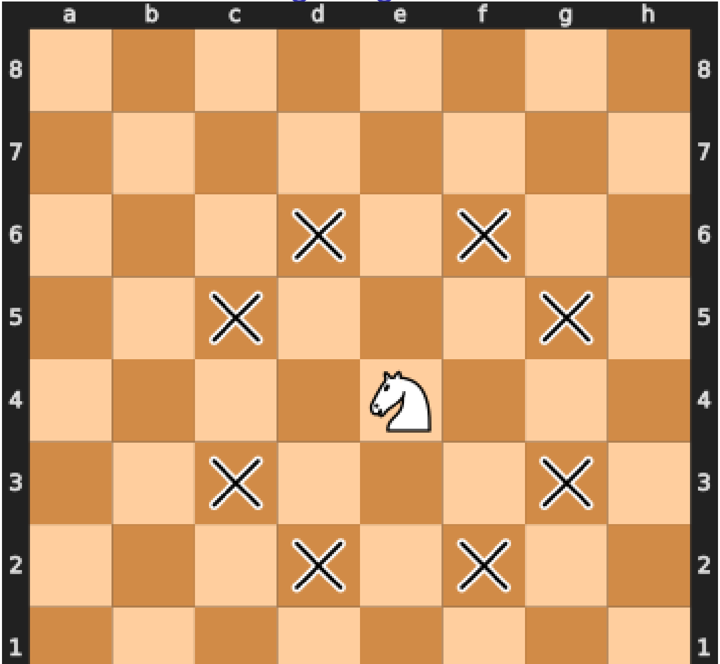
A queen is basically a bishop and a rook. Therefore we enforce both constraints. *



Answer:

- maxmimize $\sum_{ij} x_{ij}$ subject to
- $\sum_{i+j=k} x_{ij} \leq 1$ for $k = 2, \ldots, 16$.
- $\sum_{i-j=k} x_{ij} \leq 1$ for $k = -7, \ldots, 7$.
- $\sum_{i} x_{ij} \leq 1$.
- $\sum_{j} x_{ij} \leq 1$.

Question 4: Non-attacking Knights



Answer:

- Let $jump(i, j)$ be the coordinates the knight can jump from (i, j).
- Therefore we get
- maximize $\sum_{ij} x_{ij}$ subject to
- $x_{i,j} + x_{i',j'} \leq 1$ for $(i', j') \in jump(i, j)$.

Question 5: Non-attacking Kings