Integer Linear Programming

Junaid Hasan

Math 381 Lecture 3

Knapsack Problem

- Alice wants to pack a set of items into her knapsack (backpack). However she can carry at most 100 pounds. The weights and values are given below

Item	Weight	Value
1	30	240
2	35	300
3	10	100
4	15	150
5	35	360
6	22	180
7	29	220
8	18	140
9	11	90

- Which items must she pack to maximize total value if there is just one of each item.

Thoughts

- Which items seem more enticing?

Mathematical Model

- Let us introduce a variable $ x_i $ such that
- $ x_i = 0 $ if object i is not in the knapsack.
- $ x_i = 1 $ if object i is in the knapsack.
- Can you formulate the problem?
- Note that x_i is known as a *binary variable*.

(contd..)

- Then the LP is
- Maximize

$$240x_1+300x_2+100x_3+150x_4+360x_5+180x_6+220x_7+140x_8+90x_9$$

subject to

-

$$30x_1+35x_2+10x_3+15x_4+35x_5+22x_6+29x_7+18x_8+11x_9 \leq 100$$

-

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \in \{0, 1\}$$

.

- Note that the variables are constrained to be integers (more specifically binary in this case).
- This program is called an **integer linear program**.

Solution (code)

```
objects = [1,2,3,4,5,6,7,8,9]
weight = {1:30, 2:35, 3:10, 4:15, 5:35, 6:22, 7:29, 8:18, 9
value = {1:240, 2:300, 3:100, 4:150, 5:360, 6:180, 7:220, 8
maxweight = 100

solveKnapsack(objects, weight, value, maxweight)
```

Output

```
Total Price:  930.0
1 -0.0
2 -0.0
3 1.0
4 1.0
5 1.0
6 1.0
7 -0.0
8 1.0
9 -0.0
```

Remarks

- This means that Alice must put items 3,4,5,6,8.
- Moreover the total weight of these is $10+15+35+22+18 = 100$
- So the constraint is *binding* (equality is acheived).
- Let us try another experiment. Let us vary the weight capacity and see what total value she can pack.

Plot

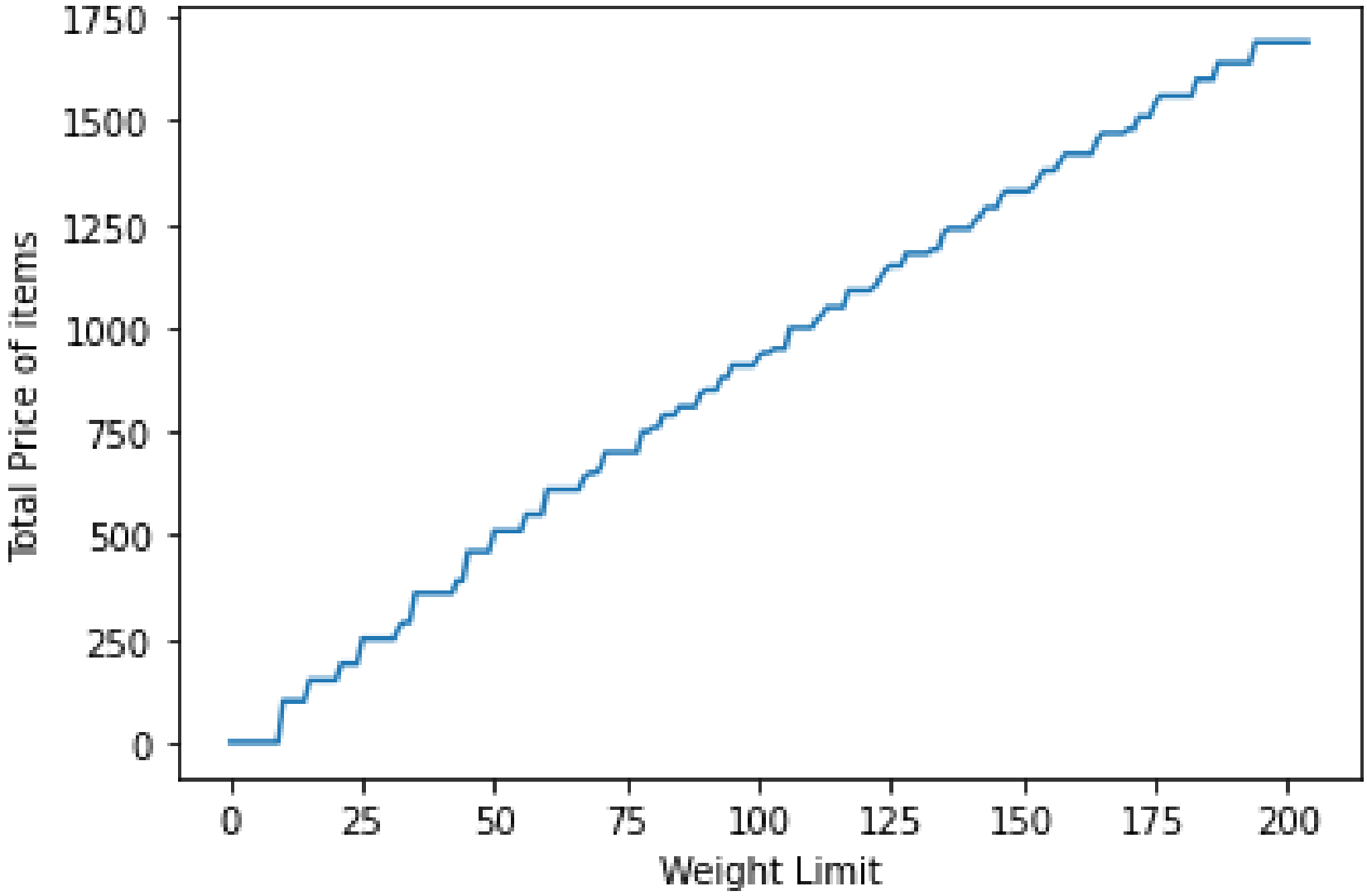

Figure 1: Value vs Size

Thoughts

- What other questions can we ask?
- Any other ideas?

Variation

- Suppose now that the objects are not limited to 1 in quantity, but rather there is an unlimited number of each.
- This can happen if its not an art show but say a grocery store.
- What should we change in the LP?
- Answer: Now $ x_i $ is not just 0 or 1, but $ x_i $ can be any non-negative integer.

Solution

- The only change we do is instead of binary

```
x[i] = model.addVar(vtype="B", name="x(%s)"%i)
```

- We have integer variables

```
x[i] = model.addVar(vtype="I", name="x(%s)"%i)
```

Remarks

- Now it seems that object 3 is picked 3 times and object 5 twice.
- Why?

Ideas

- Where can you apply the Knapsack problem idea?
- An example can be to determine the best movies to show in a film festival of a total time of 24 hours, with weights being the movie lengths and the price being the ratings (obtained from movielens).
- Similarly what songs to sing in a Karaoke.