MATH 125 **TA: Junaid Hasan**

Midterm 2 Practice

Integrals

Follow this idea:

- Look for simplification/substitution (like $\sqrt{x}$ means $u = \sqrt{x}$ or $\sqrt{4-x}$ means $u = 4-x$). Do not forget dx! For example
$$\int e^x \sin(e^x)\cos(e^x)\,dx$$
We solve this by $u = e^x$, then $du = e^x dx$. Therefore the integral becomes
$$\int \sin(u)\cos(u)\,du.$$

- Follow this guideline

Logs, Inverse Trig	BY PARTS
$\sin, \cos, \tan, \sec$	Trig
Quadratic under a radical	Trig Sub
Rational Function	Partial Fractions

Example

  - For $\sqrt{x^2-a^2}$ substitute $x = a\sec\theta$.
  - For $\sqrt{a^2-x^2}$ substitute $x = a\sin\theta$
  - For $\sqrt{a^2+x^2}$ substitute $x = a\tan\theta$.

    Note the above work even when power is 3/2 or 5/2 for example in terms like $(x^2+a^2)^{\frac{3}{2}}$.
  - Partial Fraction example:
$$\int \frac{x\,dx}{x(x+2)^2(x^2+1)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2} + \frac{Dx+E}{(x^2+1)} + \frac{Fx+G}{(x^2+1)^2}$$

Question: Evaluate the following:

1. $\int \frac{x^2+7}{x^2(3-x)}\,dx$
2. $\int \sqrt{x}\ln(x)\,dx$
3. $\int \frac{1}{(x^2+6x+13)^{3/2}}\,dx$
4. $\int \tan^{-1} x\,dx$
5. $\int \sin^2(x)\cos^3(x)\,dx$

6. $\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$
7. $\int \frac{\sqrt{x}}{x-9}\,dx$
8. $\int \tan^4 x \sec^4 x\,dx$
9. $\int x\sqrt{4-x}\,dx$
10. $\int \tan^3 x \sec x\,dx$
11. $\int x^2 \ln(x)\,dx$
12. $\int x\sqrt{5-x}\,dx$
13. $\int \frac{\sqrt{x^2-1}}{x^2}\,dx$
14. $\int \frac{x^2+1}{x^2-2x-3}\,dx$
15. $\int x \tan^{-1}(x)\,dx$
16. $\int \frac{1}{\sqrt{4x^2+8x-12}}\,dx$

Work

A well is shaped of a cylinder of radius 1 m and depth 8 m and is *half full*. Find the work done to pump all the water out. (Density of water 1000 kg/m^3 and $g = 9.8$ m/sec^2).

Arc length

Often in the problem you would need to approximate the integral using **Trapezoid Rule or Simpsons Rule**. Here is an example problem that is simpler to compute and does not require approximation.

(The formula for arc length of $f(x)$ between a and b is $\int_a^b \sqrt{1+(f'(x))^2}\,dx$.

1. Find the arc-length of the function $\ln(\cos(x))$ from $x = 0$ to $x = \frac{\pi}{3}$.

   Answer: $\ln(2+\sqrt{3}) \approx 1.317$.

All the best for the midterm!