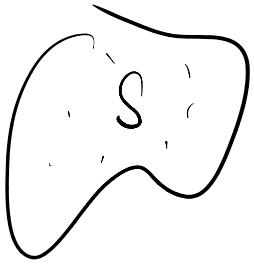
Applications of Flux

- **Charge** enclosed by a surface can be found out via flux of Electric field

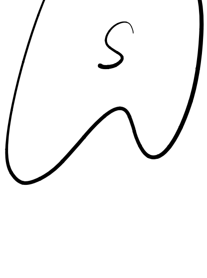
ie,

$$\frac{Q}{\varepsilon_0} = \int_S \vec{E} \cdot d\vec{S}$$



- Heat Flow field $\vec{F} = -k\nabla u$ is given by negative gradient of temperature u.

$\therefore$ Flux of $\vec{F}$: rate of heat flow across

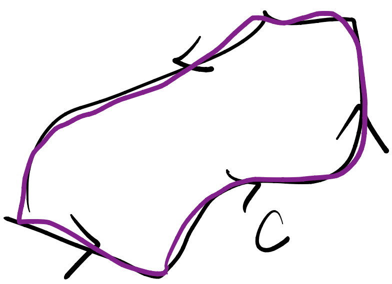


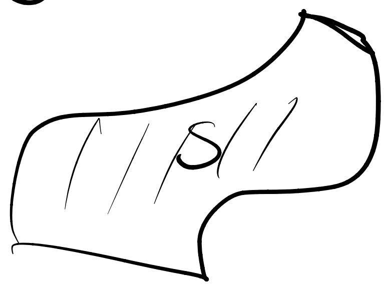
$$= \int_S \vec{F} \cdot d\vec{S}$$

$$= \int -k \nabla u \cdot d\vec{S}$$

Stokes' thm If S is a surface, C is its boundary (also denoted by ∂S) then

$$\int_C \vec{F} \cdot d\vec{r} = \int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S}$$

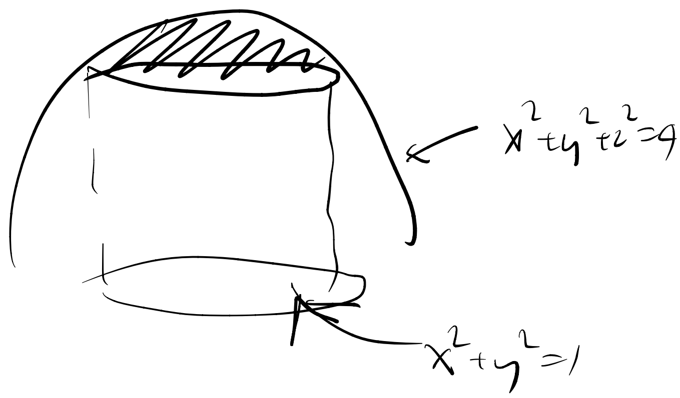




Eg Use Stokes' thm to compute $\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$

where $\vec{F}(x,y,z) = \langle xz, yz, xy \rangle$

and S =



$x^2+y^2=1$, $z=\sqrt{3}$

$\vec{r}(t) = \langle \cos t, \sin t, \sqrt{3} \rangle$ $\quad 0 \le t \le 2\pi$

$\vec{r}\,'(t) = \langle -\sin t, \cos t, 0 \rangle$

$$\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$$

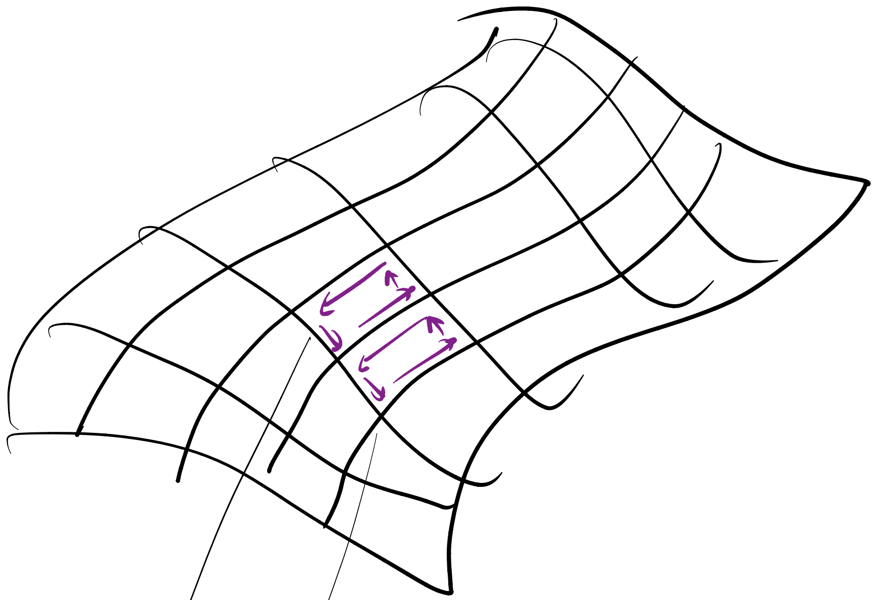
$$= \int_0^{2\pi} \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

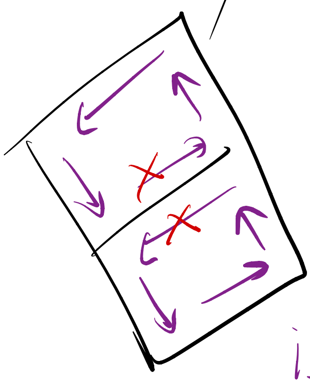
$$= \int_0^{2\pi} \langle \sqrt{3}\cos t, \sqrt{3}\sin t, \sin t \cos t \rangle \cdot \langle -\sin t, \cos t, 0 \rangle \, dt$$

$$= \int_0^{2\pi} -\sqrt{3}\sin t \cos t + \sqrt{3}\sin t \cos t \; dt$$

$$= \int_0^{2\pi} 0 \; dt = 0$$

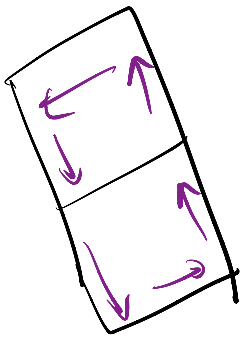
An idea of proof



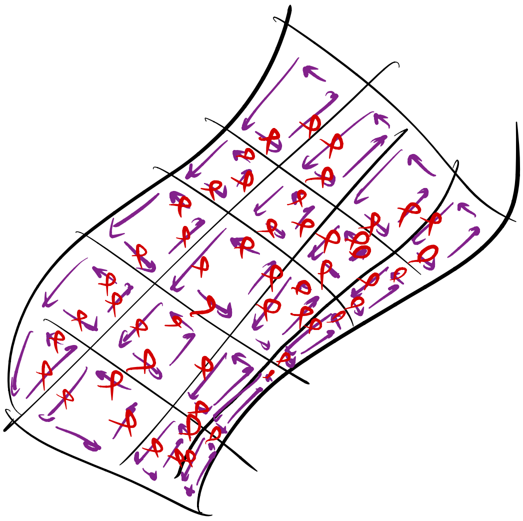


curl of $\vec{F}$ ie, $\vec{\nabla} \times \vec{F}$

is



$\therefore$ if we add consecutive pieces only the boundary term remains.



The inner arrows cancel only the outer (boundary) remains.