Computing divergence, curl

$\vec{F}(x,y,z) = e^{x}\hat{i} + yz\hat{j} - y^{2}\hat{k}$

find divergence of $\vec{F}$ at $(0,2,-1)$

Ans $= e^{0} - 1 + 4 = 4$

Finding if a field is magnetic (source free)

$\vec{F}(x,y) = \langle x^{2}y, y - xy^{2} \rangle$. Can it be magnetic?

Ans: div $\vec{F} = 0 \Leftarrow$ for source free magnetic fields.

div $\vec{F} = 0 \Rightarrow$ Source free for simply connected domain.

Is $\vec{F}(x,y) = \langle x^{2}y, 5 - xy^{2} \rangle$ source free?

Ans: Yes.

Q/ $\vec{F}(x,y) = \langle -ay, bx \rangle$ be a rotational field. Is it source free?

divergence and fluid flow

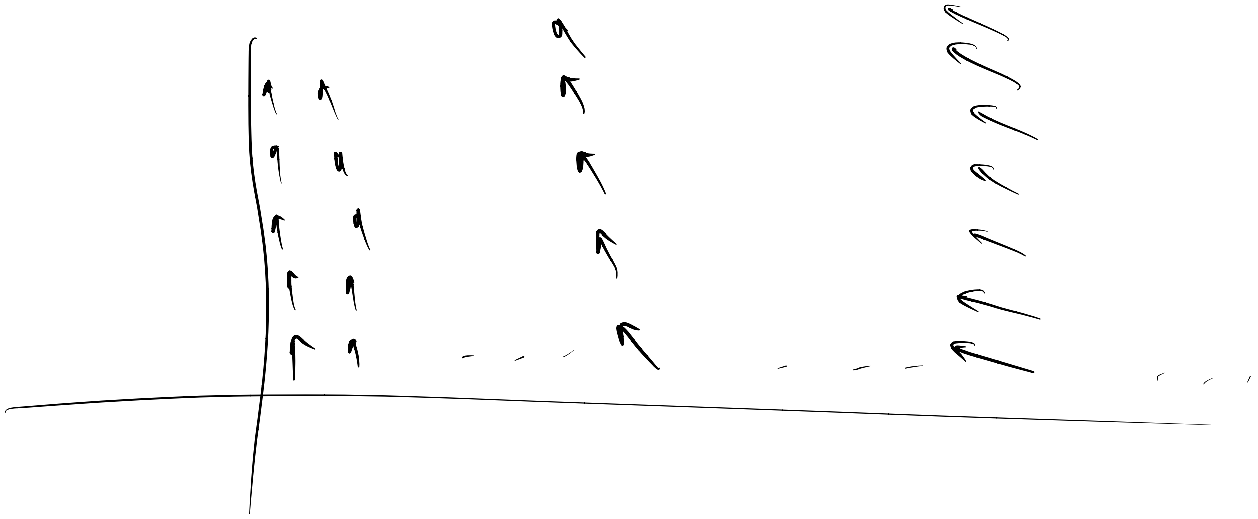
Suppose $\vec{v}(x,y) = \langle -xy, y \rangle$, $y > 0$

models fluid flow

Is more fluid flowing into $(1,4)$ than flowing out?

Ans: $\text{div}(\vec{v}) = -y + 1$

at $(1,4)$ $\quad \text{div}(\vec{v}) = -3$. $\therefore$ more fluid is flowing in



Find all points $\vec{P}$ on $v\langle x,y\rangle = \langle -xy, y\rangle$ such that the amount of fluid flowing in to P equals the amount flowing out of P.

Curl

Measures extent of rotation of the field about a point.

Divergence of the curl

$$\text{div curl}(\vec{F}) = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$$

Pf

$$\text{div}\left[(R_y - Q_z)\hat{\imath} + (P_z - R_x)\hat{\jmath} + (Q_x - P_y)\hat{k}\right]$$

$$= R_{yx} - Q_{xz} + P_{yz} - R_{yx} + Q_{zx} - P_{zy}$$

$$= 0.$$

Showing a vector field is not curl of another?

Show that $\vec{F}(x,y,z) = e^{x}\hat{i} + yz\hat{j} + xz^{2}\hat{k}$

is not the curl of another vector field.

ie, there is not any $\vec{G}$ such that $\vec{\nabla}\times\vec{G} = \vec{F}$

__Ans:__ If it were true then

$$\text{div}(\text{curl}\,\vec{G}) = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{G})$$
$$= 0$$

$$\Rightarrow \quad \vec{\nabla}\cdot(\vec{F}) = 0$$

$$\text{But } \vec{\nabla}\cdot\vec{F} = e^{x} + z + 2xz$$
$$\neq 0,$$

Thm Curl of a conservative vector field $\vec{\nabla} \times (\vec{F}) = 0$

for $\vec{F} = \vec{\nabla} f$

$$\text{curl } \vec{F} = (R_y - Q_z)\hat{i} + (P_z - R_x)\hat{j} + (Q_x - P_y)\hat{k}$$

But for conservative vector fields

$R_y = Q_z$, $P_z = R_x$, $Q_x = P_y$.

We saw that Conservative $\Rightarrow$ curl $\vec{F} = 0$

But suppose curl $\vec{F} = 0$

then is it true that $\vec{F}$ is conservative ?

Ans : Yes If domain is simply connected .

Determine if $\vec{F}(x,y,z) = \langle yz, xz, xy \rangle$

is conservative.

Ans: The domain $\vec{F}$ is all of $\mathbb{R}^3$

which is simply connected.

$\therefore$ We can test $\vec{F}$ is conservative by

computing the curl.