Math 224 B **EXAM 2** May 25, 2022

Name ______________________ UW ID: ______________________

Academic Honesty Statement: All work on this exam is my own.

SIGNATURE:

Instructions:

- Your exam should consist of this cover sheet, followed by 6 problems, and scratch paper.
- Check that on your exam the bottom of the last page says **END OF EXAM.**
- The points for each question are indicated at the beginning of each question.
- Pace yourself. You have 50 minutes to complete the exam and there are 6 problems. Total marks is 50. **If you score more than 50, you get 50.**
- **Show all your work**, unless the problems says otherwise explicitly. An answer without work shown will receive little or no credit.
- Please place a **box around your final answer** to each question.
- If you need more space for your answer, use the back of the page and indicate that you have done so.
- Your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use a scientific calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- Cheating will result in a **zero** and will be reported to the Dean's Academic Conduct Committee.
- Time allowed: 50 minutes.

GOOD LUCK!

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	50	

1. (10 points, 5 each)

   (a) Is there a vector field $\mathbf{G}$ such that

   $$\operatorname{curl}\mathbf{G} = \langle 2x, 3yz, -xz^2 \rangle.$$

   If the answer is **yes**, then find $\mathbf{G}$; else if the answer is **no**, explain why?

   (b) Is there a function $f(x, y, z)$ such that

   $$\operatorname{grad} f = \langle y, -x, e^z \rangle.$$

   If the answer is **yes**, then find f; else if the answer is **no**, explain why?

2. A force defined in 3-space has the form $\mathbf{F}(x, y, z) = \langle y, z, yz \rangle$.

   (a) (4 points) Is $\mathbf{F}$ conservative? Explain.

   (b) (6 points) Calculate the work done by this force in moving a particle along a curve

   $$\mathbf{r}(t) = \langle \cos t, \sin t, e^t \rangle$$

   from $(1, 0, 1)$ to $(-1, 0, e^{\pi})$.

3. (10 points) Evaluate

$$\oint_C y^2\,dx + x\,dy$$

where C is the curve counterclockwise along the boundary of a square of side 2 centered at origin. To be precise C is the curve formed by straight lines from (1,-1) to (1,1) to (-1,1) to (-1,-1) and back to (1,-1).

4. (10 points) Compute the area of that portion of the surface

$$z^2 = 2xy$$

in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ between the planes $x = 0, x = 4$ and $y = 0, y = 1$.

5. (10 points, 5 each) The function g of three variables is given by $g(x, y, z) = xz^2 + y - e^6$.

   (a) Suppose $\mathbf{r}(t)$ is a parameterized curve; we do not know the formula for $\mathbf{r}(t)$, but we know that $\mathbf{r}(5) = \langle 1, 2, 3\rangle$ and $\mathbf{r}'(5) = \langle -1, \pi, 2\rangle$. Define a new function $h(t) = g(\mathbf{r}(t))$; find $h'(5)$.

   (b) Suppose you are at the point $(1, 2, 3)$ and you want to start moving in a direction so that g stays constant. Give one possible direction for which this is true.

6. (10 points) Let S be the surface parameterized by

$$\mathbf{r}(u,v) = \langle 2u+v, 4u+3v, u^2\rangle, \qquad \text{with } 0 \le u \le 1, \quad 0 \le v \le 1,$$

oriented downward (the normal to the surface has negative z-coordinate), and let C be its boundary with the compatible orientation. Evaluate

$$\oint_C \mathbf{F} \cdot d\,\mathbf{r}$$

where

$$\mathbf{F}(x,y,z) = \langle 2z + \sin(x), 3x + \cos y, y\rangle.$$

END OF EXAM

SCRATCH PAPER

END OF EXAM