Math 224 B **MIDTERM EXAM 1** April 20, 2022

Name _____ UW ID: _____

Academic Honesty Statement: All work on this exam is my own.

SIGNATURE:

Instructions:

- Your exam should consist of this cover sheet, followed by 5 problems, and scratch paper.
- Check that on your exam the bottom of the last page says **END OF EXAM.**
- The points for each question are indicated at the beginning of each question.
- Pace yourself. You have 50 minutes to complete the exam and there are 5 problems. Try not to spend more than 10 minutes on each page.
- **Show all your work**, unless the problems says otherwise explicitly. An answer without work shown will receive little or no credit.
- Please place a **box around your final answer** to each question.
- If you need more space for your answer, use the back of the page and indicate that you have done so.
- Your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use a scientific calculator and one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing calculators) are forbidden.
- Cheating will result in a **zero** and will be reported to the Dean's Academic Conduct Committee.
- Time allowed: 50 minutes.

GOOD LUCK!

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points, 2 points each) State **True/False**. You **don't need** to provide justification.

(a) $\int_0^1 \int_1^2 \sqrt{1+x^2}\,dx\,dy = \int_1^2 \int_0^1 \sqrt{1+x^2}\,dy\,dx.$

(b) $\int_0^x \int_1^2 \sqrt{1+x^2}\,dx\,dy = \int_1^2 \int_0^x \sqrt{1+x^2}\,dy\,dx.$

(c) $\int_1^2 \int_3^4 x^2 e^{y^2}\,dydx = \left[\int_1^2 x^2\,dx\right] \cdot \left[\int_3^4 e^{y^2}\,dy\right].$

(d) $\int_1^2 \int_3^x x^2 e^{y^2}\,dydx = \left[\int_1^2 x^2\,dx\right] \cdot \left[\int_3^x e^{y^2}\,dy\right].$

(e) $\int_0^2 \int_0^{\sqrt{x}} \sqrt{y} e^{y^2}\,dy\,dx = \int_0^2 \int_0^{\sqrt{y}} \sqrt{x} e^{x^2}\,dx\,dy.$

2. When a double integral was set up for the volume V of the solid under the surface $z = f(x, y)$ and above a region S of the xy-plane, the following sum of iterated integrals was obtained:

$$V = \int_1^2 \left[\int_x^{x^2} f(x, y)\, dy \right] dx + \int_2^4 \left[\int_x^4 f(x, y)\, dy \right] dx.$$

(a) (4 points) Sketch the region S and express V as an iterated integral in which the order of integration is reversed.

(b) (6 points) Carry out the integration and compute V when $f(x, y) = \frac{16x(x^2+1)}{y}$.

3. (10 points) Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

4. (10 points) A solid is bounded by two concentric hemispheres of radii a and b, where $0 < a < b$. Find the center of mass if the density at a point is equal to the square of the distance of this point from the center.

5. (10 points) Use a suitable change of variable to show that

$$\iint_S f(x+y)\,dx\,dy = \int_{-1}^{1} f(u)\,du,$$

where $S = \{(x, y) \quad | \quad |x| + |y| \leq 1\}$.

END OF EXAM

SCRATCH PAPER

END OF EXAM